ALGEBRA 2

SUMMER PACKET 2020 – 2021

CONWELL-EGAN CATHOLIC HIGH SCHOOL

All students who are enrolled in a mathematics course for the 2020 – 2021 school year have a Mathematics Summer Packet to complete.

Within the first few days of your Algebra 2 course, you will be assessed on the prerequisite skills outline in this packet. This summer assignment is a review and exploration of key skills that are necessary for success in your 2020 – 2021 mathematics course as well as future high school mathematics courses.

The assessment will count as a full test grade in your first quarter average.

All summer packets are due on September 14, 2020

EXPRESSIONS AND FORMULAS

EXAMPLE 1 Evaluate Algebraic Expressions

a. Evaluate $m + (n - 1)^2$ if $m = 3$ and $n = -4$.

$m + (n - 1)^2 = 3 + (-4 - 1)^2$ Replace m with 3 and n with -4.

$= 3 + (-5)^2$ Add -4 and -1.

$= 3 + 25$ Find $(-5)^2$.

$= 28$ Add 3 and 25.

b. Evaluate $x^2 - y(x + y)$ if $x = 8$ and $y = 1.5$.

$x^2 - y(x + y) = 8^2 - 1.5(8 + 1.5)$ Replace x with 8 and y with 1.5.

$= 8^2 - 1.5(9.5)$ Add 8 and 1.5.

$= 64 - 1.5(9.5)$ Find 8^2.

$= 64 - 14.25$ Multiply 1.5 and 9.5.

$= 49.75$ Subtract 14.25 from 64.

c. Evaluate $\frac{a^3 + 2bc}{c^2 - 5}$ if $a = 2$, $b = -4$, and $c = -3$.

$\frac{a^3 + 2bc}{c^2 - 5} = \frac{2^3 + 2(-4)(-3)}{(-3)^2 - 5}$ $a = 2$, $b = -4$, and $c = -3$

$= \frac{8 + (-8)(-3)}{9 - 5}$ Evaluate the numerator and the denominator separately.

$= \frac{8 + 24}{9 - 5}$ Multiply -8 by -3.

$= \frac{32}{4}$ or 8 Simplify the numerator and the denominator. Then divide.

Evaluate each expression if $q = \frac{1}{2}$, $r = 1.2$, $s = -6$, and $t = 5$.

1. $qr - st$
2. $qr \div st$
3. $qrst$
4. $qr + st$
5. $\frac{3q}{4s}$
6. $\frac{5qr}{t}$
7. $\frac{2r(4s - 1)}{t}$
8. $\frac{4q^3s + 1}{t - 1}$

Evaluate each expression if $a = -0.5$, $b = 4$, $c = 5$, and $d = -3$.

9. $3b + 4d$
10. $ab^2 + c$
11. $bc + d \div a$
12. $7ab - 3d$
13. $ad + b^2 - c$
14. $\frac{4a + 3c}{3b}$
15. $\frac{3ab^2 - d^3}{a}$
16. $\frac{5a + ad}{bc}$

PROPERTIES OF REAL NUMBERS

EXAMPLE Classify Numbers

1 Name the sets of numbers to which each number belongs.

a. $\sqrt{16}$

$\sqrt{16} = 4$ naturals (N), wholes (W), integers (Z), rationals (Q), reals (R)

b. -18 integers (Z), rationals (Q), and reals (R)

c. $\sqrt{20}$ irrationals (I) and reals (R)

$\sqrt{20}$ lies between 4 and 5 so it is not a whole number.

d. $-\frac{7}{8}$ rationals (Q) and reals (R)

e. $0.\overline{45}$ rationals (Q) and reals (R)

The bar over the 45 indicates that those digits repeat forever.

EXAMPLE Identify Properties of Real Numbers

2 Name the property illustrated by $(5 + 7) + 8 = 8 + (5 + 7)$.

Commutative Property of Addition

The Commutative Property says that the order in which you add does not change the sum.

EXAMPLE Simplify an Expression

5 Simplify $2(5m + n) + 3(2m - 4n)$.

$2(5m + n) + 3(2m - 4n)$

$= 2(5m) + 2(n) + 3(2m) - 3(4n)$ Distributive Property

$= 10m + 2n + 6m - 12n$ Multiply.

$= 10m + 6m + 2n - 12n$ Commutative Property (+)

$= (10 + 6)m + (2 - 12)n$ Distributive Property

$= 16m - 10n$ Simplify.

KEY CONCEPT — Real Number Properties

For any real numbers a, b, and c:

Property	Addition	Multiplication
Commutative	$a + b = b + a$	$a \cdot b = b \cdot a$
Associative	$(a + b) + c = a + (b + c)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Identity	$a + 0 = a = 0 + a$	$a \cdot 1 = 1 \cdot a$
Inverse	$a + (-a) = 0 = (-a) + a$	If $a \neq 0$, then $a \cdot \frac{1}{a} = 1 = \frac{1}{a} \cdot a$.
Distributive	$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$	

Name the sets of numbers to which each number belongs. (Use N, W, Z, Q, I, and R.)

1. 8.2
2. -9
3. $\sqrt{36}$
4. $-\frac{1}{3}$
5. $\sqrt{2}$
6. $-0.\overline{24}$

Name the property illustrated by each equation.

7. $(4 + 9a)2b = 2b(4 + 9a)$
8. $3\left(\frac{1}{3}\right) = 1$
9. $a(3 - 2) = a \cdot 3 - a \cdot 2$
10. $(-3b) + 3b = 0$
11. $jk + 0 = jk$
12. $(2a)b = 2(ab)$

Simplify each expression.

13. $7s + 9t + 2s - 7t$
14. $6(2a + 3b) + 5(3a - 4b)$
15. $4(3x - 5y) - 8(2x + y)$
16. $0.2(5m - 8) + 0.3(6 - 2m)$
17. $\frac{1}{2}(7p + 3q) + \frac{3}{4}(6p - 4q)$
18. $\frac{4}{5}(3v - 2w) - \frac{1}{5}(7v - 2w)$

SOLVING EQUATIONS

EXAMPLE 1 Verbal to Algebraic Expression

Write an algebraic expression to represent each verbal expression.

a. **three times the square of a number** $3x^2$

b. **twice the sum of a number and 5** $2(y + 5)$

EXAMPLE 2 Algebraic to Verbal Sentence

Write a verbal sentence to represent each equation.

a. $n + (-8) = -9$ The sum of a number and -8 is -9.

b. $\frac{n}{6} = n^2$ A number divided by 6 is equal to that number squared.

EXAMPLE 3 Identify Properties of Equality

Name the property illustrated by each statement.

a. **If $3m = 5n$ and $5n = 10p$, then $3m = 10p$.**

Transitive Property of Equality

b. **If $12m = 24$, then $(2 \cdot 6)m = 24$.**

Substitution

KEY CONCEPT *Properties of Equality*

Property	Symbols	Examples
Reflexive	For any real number a, $a = a$.	$-7 + n = -7 + n$
Symmetric	For all real numbers a and b, if $a = b$, then $b = a$.	If $3 = 5x - 6$, then $5x - 6 = 3$.
Transitive	For all real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.	If $2x + 1 = 7$ and $7 = 5x - 8$, then $2x + 1 = 5x - 8$.
Substitution	If $a = b$, then a may be replaced by b and b may be replaced by a.	If $(4 + 5)m = 18$, then $9m = 18$.

EXAMPLE Solve One-Step Equations

4 **Solve each equation. Check your solution.**

a. $a + 4.39 = 76$

$$a + 4.39 = 76 \quad \text{Original equation}$$

$$a + 4.39 - 4.39 = 76 - 4.39 \quad \text{Subtract 4.39 from each side.}$$

$$a = 71.61 \quad \text{Simplify.}$$

The solution is 71.61.

CHECK

$$a + 4.39 = 76 \quad \text{Original equation}$$

$$71.61 + 4.39 \stackrel{?}{=} 76 \quad \text{Substitute 71.61 for } a.$$

$$76 = 76 \checkmark \quad \text{Simplify.}$$

b. $-\frac{3}{5}d = 18$

$$-\frac{3}{5}d = 18 \quad \text{Original equation}$$

$$-\frac{5}{3}\left(-\frac{3}{5}\right)d = -\frac{5}{3}(18) \quad \text{Multiply each side by } -\tfrac{5}{3}\text{, the multiplicative inverse of } -\tfrac{3}{5}.$$

$$d = -30 \quad \text{Simplify.}$$

The solution is -30.

CHECK

$$-\frac{3}{5}d = 18 \quad \text{Original equation}$$

$$-\frac{3}{5}(-30) \stackrel{?}{=} 18 \quad \text{Substitute } -30 \text{ for } d.$$

$$18 = 18 \checkmark \quad \text{Simplify.}$$

EXAMPLE Solve a Multi-Step Equation

5 **Solve $2(2x + 3) - 3(4x - 5) = 22$.**

$$2(2x + 3) - 3(4x - 5) = 22 \quad \text{Original equation}$$

$$4x + 6 - 12x + 15 = 22 \quad \text{Apply the Distributive Property.}$$

$$-8x + 21 = 22 \quad \text{Simplify the left side.}$$

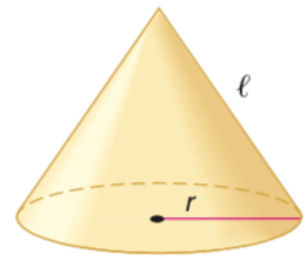
$$-8x = 1 \quad \text{Subtract 21 from each side to isolate the variable.}$$

$$x = -\frac{1}{8} \quad \text{Divide each side by } -8.$$

The solution is $-\frac{1}{8}$.

EXAMPLE Solve for a Variable

6 **GEOMETRY** **The formula for the surface area S of a cone is $S = \pi r\ell + \pi r^2$, where ℓ is the slant height of the cone and r is the radius of the base. Solve the formula for ℓ.**



$S = \pi r\ell + \pi r^2$	Surface area formula
$S - \pi r^2 = \pi r\ell + \pi r^2 - \pi r^2$	Subtract πr^2 from each side.
$S - \pi r^2 = \pi r\ell$	Simplify.
$\dfrac{S - \pi r^2}{\pi r} = \dfrac{\pi r\ell}{\pi r}$	Divide each side by πr.
$\dfrac{S - \pi r^2}{\pi r} = \ell$	Simplify.

KEY CONCEPT — *Properties of Equality*

Addition and Subtraction

Symbols For any real numbers a, b, and c, if $a = b$, then $a + c = b + c$ and $a - c = b - c$.

Examples If $x - 4 = 5$, then $x - 4 + 4 = 5 + 4$.
If $n + 3 = -11$, then $n + 3 - 3 = -11 - 3$.

Multiplication and Division

Symbols For any real numbers a, b, and c, if $a = b$, then $a \cdot c = b \cdot c$, and if $c \neq 0$, $\frac{a}{c} = \frac{b}{c}$.

Examples If $\frac{m}{4} = 6$, then $4 \cdot \frac{m}{4} = 4 \cdot 6$. If $-3y = 6$, then $\frac{-3y}{-3} = \frac{6}{-3}$.

Write an algebraic expression to represent each verbal expression.

1. twelve decreased by the square of a number
2. twice the sum of a number and negative nine
3. the product of the square of a number and 6
4. the square of the sum of a number and 11

Name the property illustrated by each statement.

5. If $a + 1 = 6$, then $3(a + 1) = 3(6)$.
6. If $x + (4 + 5) = 21$, then $x + 9 = 21$.
7. If $7x = 42$, then $7x - 5 = 42 - 5$.
8. If $3 + 5 = 8$ and $8 = 2 \cdot 4$, then $3 + 5 = 2 \cdot 4$.

Solve each equation. Check your solution.

9. $5t + 8 = 88$
10. $27 - x = -4$
11. $\frac{3}{4}y = \frac{2}{3}y + 5$
12. $8s - 3 = 5(2s + 1)$
13. $3(k - 2) = k + 4$
14. $0.5z + 10 = z + 4$
15. $8q - \frac{q}{3} = 46$
16. $-\frac{2}{7}r + \frac{3}{7} = 5$
17. $d - 1 = \frac{1}{2}(d - 2)$

Solve each equation or formula for the specified variable.

18. $C = \pi r$; for r
19. $I = Prt$, for t
20. $m = \frac{n - 2}{n}$, for n

SOLVING ABSOLUTE-VALUE EQUATIONS

EXAMPLE Evaluate an Expression with Absolute Value

1 **Evaluate $1.4 + |5y - 7|$ if $y = -3$.**

$$
\begin{aligned}
1.4 + |5y - 7| &= 1.4 + |5(-3) - 7| && \text{Replace } y \text{ with } -3. \\
&= 1.4 + |-15 - 7| && \text{Simplify } 5(-3) \text{ first.} \\
&= 1.4 + |-22| && \text{Subtract 7 from } -15. \\
&= 1.4 + 22 && |-22| = 22 \\
&= 23.4 && \text{Add.}
\end{aligned}
$$

EXAMPLE 2 Solve an Absolute Value Equation

Solve $|x - 18| = 5$. Check your solutions.

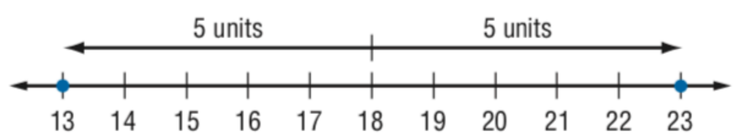
Case 1	$a = b$	or	**Case 2**	$a = -b$
	$x - 18 = 5$			$x - 18 = -5$
	$x - 18 + 18 = 5 + 18$			$x - 18 + 18 = -5 + 18$
	$x = 23$			$x = 13$

CHECK

$\|x - 18\| = 5$	$\|x - 18\| = 5$
$\|23 - 18\| \stackrel{?}{=} 5$	$\|13 - 18\| \stackrel{?}{=} 5$
$\|5\| \stackrel{?}{=} 5$	$\|-5\| \stackrel{?}{=} 5$
$5 = 5$ ✓	$5 = 5$ ✓

The solutions are 23 and 13. Thus, the solution set is {13, 23}.

On the number line, we can see that each answer is 5 units away from 18.



EXAMPLE 3 No Solution

Solve $|5x - 6| + 9 = 0$.

$|5x - 6| + 9 = 0$ Original equation

$|5x - 6| = -9$ Subtract 9 from each side.

This sentence is *never* true. So the solution set is ∅.

EXAMPLE 4 One Solution

Solve $|x + 6| = 3x - 2$. Check your solutions.

Case 1 $\quad a = b$ $\quad$ or $\quad$ **Case 2** $\quad a = -b$

$$\begin{aligned} x + 6 &= 3x - 2 \\ 6 &= 2x - 2 \\ 8 &= 2x \\ 4 &= x \end{aligned} \qquad\qquad \begin{aligned} x + 6 &= -(3x - 2) \\ x + 6 &= -3x + 2 \\ 4x + 6 &= 2 \\ 4x &= -4 \\ x &= -1 \end{aligned}$$

There appear to be two solutions, 4 and -1.

CHECK Substitute each value in the original equation.

$$\begin{aligned} |x + 6| &= 3x - 2 \\ |4 + 6| &\stackrel{?}{=} 3(4) - 2 \\ |10| &\stackrel{?}{=} 12 - 2 \\ 10 &= 10 \ \checkmark \end{aligned} \qquad\qquad \begin{aligned} |x + 6| &= 3x - 2 \\ |-1 + 6| &\stackrel{?}{=} 3(-1) - 2 \\ |5| &\stackrel{?}{=} -3 - 2 \\ \cancel{5} &\cancel{= -5} \end{aligned}$$

Since $5 \neq -5$, the only solution is 4. Thus, the solution set is $\{4\}$.

Evaluate each expression if $x = -5$, $y = 3$, and $z = -2.5$.

1. $|2x|$
2. $|-3y|$
3. $|2x + y|$
4. $|y + 5z|$
5. $-|x + z|$
6. $8 - |5y - 3|$
7. $2|x| - 4|2 + y|$
8. $|x + y| - 6|z|$

Solve each equation. Check your solutions.

9. $|d + 1| = 7$

12. $|t + 9| - 8 = 5$

15. $2|y + 4| = 14$

18. $|2c + 3| - 15 = 0$

21. $2|2d - 7| + 1 = 35$

24. $|4y - 5| + 4 = 7y + 8$

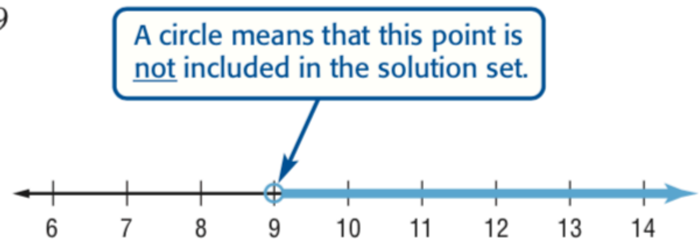
SOLVING INEQUALITIES

EXAMPLE Solve an Inequality Using Addition or Subtraction

1 Solve $7x - 5 > 6x + 4$. Graph the solution set on a number line.

$7x - 5 > 6x + 4$	Original inequality
$7x - 5 + (-6x) > 6x + 4 + (-6x)$	Add $-6x$ to each side.
$x - 5 > 4$	Simplify.
$x - 5 + 5 > 4 + 5$	Add 5 to each side.
$x > 9$	Simplify.

Any real number greater than 9 is a solution of this inequality. The graph of the solution set is shown at the right.



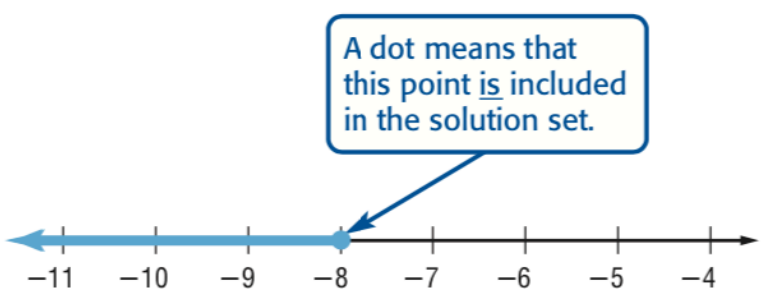
CHECK Substitute a number greater than 9 for x in $7x - 5 > 6x + 4$. The inequality should be true.

EXAMPLE Solve an Inequality Using Multiplication or Division

2 Solve $-0.25y \geq 2$. Graph the solution set on a number line.

$-0.25y \geq 2$	Original inequality
$\dfrac{-0.25y}{-0.25} \leq \dfrac{2}{-0.25}$	Divide each side by -0.25, reversing the inequality symbol.
$y \leq -8$	Simplify.

The solution set is $\{y \mid y \leq -8\}$. The graph of the solution set is shown below.



EXAMPLE Solve a Multi-Step Inequality

3 **Solve $-m \leq \frac{m+4}{9}$. Graph the solution set on a number line.**

$-m \leq \frac{m+4}{9}$ Original inequality

$-9m \leq m + 4$ Multiply each side by 9.

$-10m \leq 4$ Add $-m$ to each side.

$m \geq -\frac{4}{10}$ Divide each side by -10, reversing the inequality symbol.

$m \geq -\frac{2}{5}$ Simplify.

The solution set is $\left\{m \mid m \geq -\frac{2}{5}\right\}$ and is graphed below.

Real-World EXAMPLE Write an Inequality

4 **DELIVERIES Craig is delivering boxes of paper. Each box weighs 64 pounds, and Craig weighs 160 pounds. If the maximum capacity of the elevator is 2000 pounds, how many boxes can Craig safely take on each trip?**

Explore Let b = the number of boxes Craig can safely take on each trip. A maximum capacity of 2000 pounds means that the total weight must be less than or equal to 2000.

Plan The total weight of the boxes is $64b$. Craig's weight plus the total weight of the boxes must be less than or equal to 2000. Write an inequality.

Craig's weight	plus	the weight of the boxes	is less than or equal to	2000.
160	$+$	$64b$	$\leq$	2000

Solve $160 + 64b \leq 2000$ Original inequality

$64b \leq 1840$ Subtract 160 from each side.

$b \leq 28.75$ Divide each side by 64.

Check Since Craig cannot take a fraction of a box, he can take no more than 28 boxes per trip and still meet the safety requirements.

Solve each inequality. Then graph the solution set on a number line.

1. $2z + 5 \leq 7$
4. $-3x > 6$
6. $-33 > 5g + 7$
7. $-3(y - 2) \geq -9$
9. $5(b - 3) \leq b - 7$
10. $3(2x - 5) < 5(x - 4)$
12. $2(d + 4) - 5 \geq 5(d + 3)$
13. $8 - 3t < 4(3 - t)$
15. $\frac{a + 8}{4} \leq \frac{7 + a}{3}$
16. $-y < \frac{y + 5}{2}$
18. $6s - (4s + 7) > 5 - s$

Define a variable and write an inequality for each problem. Then solve.

19. The product of 7 and a number is greater than 42.
20. The difference of twice a number and 3 is at most 11.
21. The product of -10 and a number is greater than or equal to 20.
22. Thirty increased by a number is less than twice the number plus three.

SOLVING COMPOUND AND ABSOLUTE-VALUE INEQUALITIE

EXAMPLE 1 Solve an "and" Compound Inequality

Solve $13 < 2x + 7 \leq 17$. Graph the solution set on a number line.

Method 1

Write the compound inequality using the word *and*. Then solve each inequality.

$$\begin{aligned} 13 &< 2x + 7 \quad \text{and} \quad & 2x + 7 &\leq 17 \\ 6 &< 2x & 2x &\leq 10 \\ 3 &< x & x &\leq 5 \end{aligned}$$

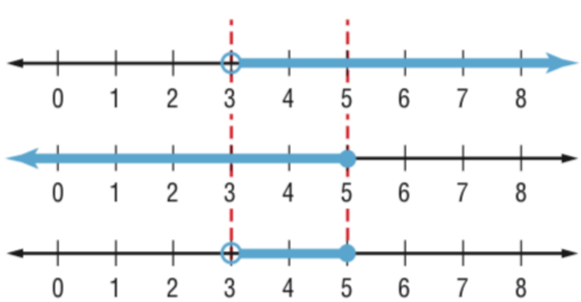
$$3 < x \leq 5$$

Method 2

Solve both parts at the same time by subtracting 7 from each part. Then divide each part by 2.

$$\begin{aligned} 13 < 2x &+ 7 \leq 17 \\ 6 < 2x &\leq 10 \\ 3 < x &\leq 5 \end{aligned}$$

Graph the solution set for each inequality and find their intersection.



$x > 3$

$x \leq 5$

$3 < x \leq 5$

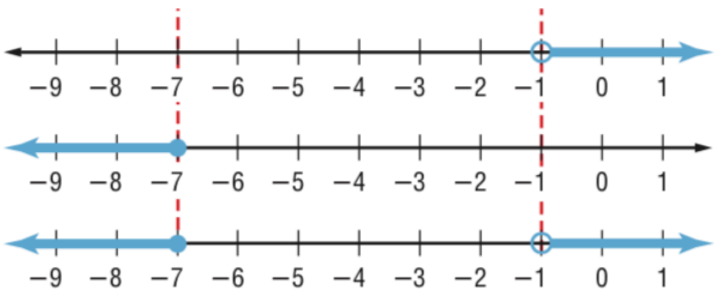
The solution set is $\{x | 3 < x \leq 5\}$.

EXAMPLE 2 Solve an "or" Compound Inequality

Solve $y - 2 > -3$ or $y + 4 \leq -3$. Graph the solution set on a number line.

Solve each inequality separately.

$$y - 2 > -3 \qquad \text{or} \qquad y + 4 \leq -3$$
$$y > -1 \qquad\qquad\qquad y \leq -7$$

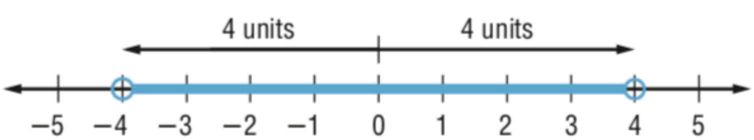


The solution set is $\{y | y > -1 \text{ or } y \leq -7\}$.

EXAMPLE 3 Solve an Absolute Value Inequality (<)

Solve $|a| < 4$. Graph the solution set on a number line.

$|a| < 4$ means that the distance between a and 0 on a number line is less than 4 units. To make $|a| < 4$ true, substitute numbers for a that are fewer than 4 units from 0.



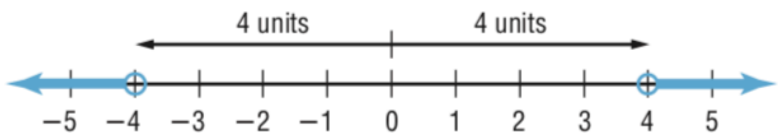
Notice that the graph of $|a| < 4$ is the same as the graph of $a > -4$ and $a < 4$.

All of the numbers between -4 and 4 are less than 4 units from 0. The solution set is $\{a | -4 < a < 4\}$.

EXAMPLE Solve an Absolute Value Inequality (>)

4 **Solve $|a| > 4$. Graph the solution set on a number line.**

$|a| > 4$ means that the distance between a and 0 on a number line is greater than 4 units.



Notice that the graph of $|a| > 4$ is the same as the graph of $\{a > 4$ *or* $a < -4\}$.

The solution set is $\{a \mid a > 4 \text{ or } a < -4\}$.

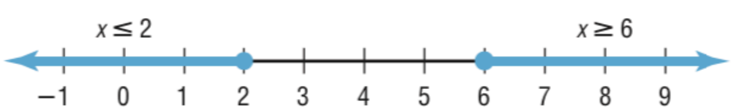
EXAMPLE Solve a Multi-Step Absolute Value Inequality

5 **Solve $|3x - 12| \geq 6$. Graph the solution set on a number line.**

$|3x - 12| \geq 6$ is equivalent to $3x - 12 \geq 6$ or $3x - 12 \leq -6$. Solve the inequality.

$3x - 12 \geq 6$	or	$3x - 12 \leq -6$	Rewrite the inequality.
$3x \geq 18$		$3x \leq 6$	Add 12.
$x \geq 6$		$x \leq 2$	Divide by 3.

The solution set is $\{x \mid x \geq 6 \text{ or } x \leq 2\}$.



Real-World EXAMPLE — Write an Absolute Value Inequality

6 **JOB HUNTING** **To prepare for a job interview, Megan researches the position's requirements and pay. She discovers that the average starting salary for the position is \$38,500, but her actual starting salary could differ from the average by as much as \$2450.**

a. Write an absolute value inequality to describe this situation.
Let x equal Megan's starting salary.

Her starting salary could differ from the average	by as much as	\$2450.
$\lvert 38{,}500 - x \rvert$	$\leq$	2450

b. Solve the inequality to find the range of Megan's starting salary.
Rewrite the absolute value inequality as a compound inequality. Then solve for x.

$$-2450 \leq 38{,}500 - x \leq 2450$$
$$-2450 - 38{,}500 \leq 38{,}500 - x - 38{,}500 \leq 2450 - 38{,}500$$
$$-40{,}950 \leq -x \leq -36{,}050$$
$$40{,}950 \geq x \geq 36{,}050$$

The solution set is $\{x \mid 36{,}050 \leq x \leq 40{,}950\}$. Thus, Megan's starting salary will fall within \$36,050 and \$40,950.

Write an absolute value inequality for each of the following. Then graph the solution set on a number line.

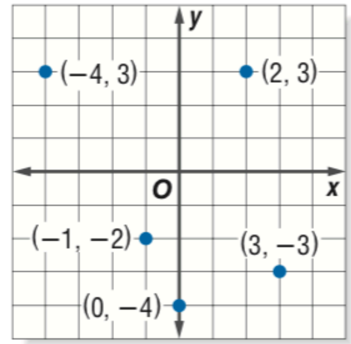
1. all numbers less than -9 and greater than 9
2. all numbers between -5.5 and 5.5
3. all numbers greater than or equal to -2 and less than or equal to 2

Solve each inequality. Graph the solution set on a number line.

4. $3m - 2 < 7$ or $2m + 1 > 13$
6. $-3 \leq s - 2 \leq 5$
8. $7 \leq 4x + 3 \leq 19$
9. $4x + 7 < 5$ or $2x - 4 > 12$
10. $\lvert 7x \rvert \geq 21$
13. $\lvert a + 3 \rvert < 1$
16. $\lvert 3d + 6 \rvert \geq 3$
19. $\lvert 2r + 4 \rvert < 6$
22. $12 + \lvert 2q \rvert < 0$

EXAMPLE Domain and Range

1 **State the domain and range of the relation shown in the graph. Is the relation a function?**



The relation is $\{(-4, 3), (-1, -2), (0, -4), (2, 3), (3, -3)\}$.
The domain is $\{-4, -1, 0, 2, 3\}$.
The range is $\{-4, -3, -2, 3\}$.

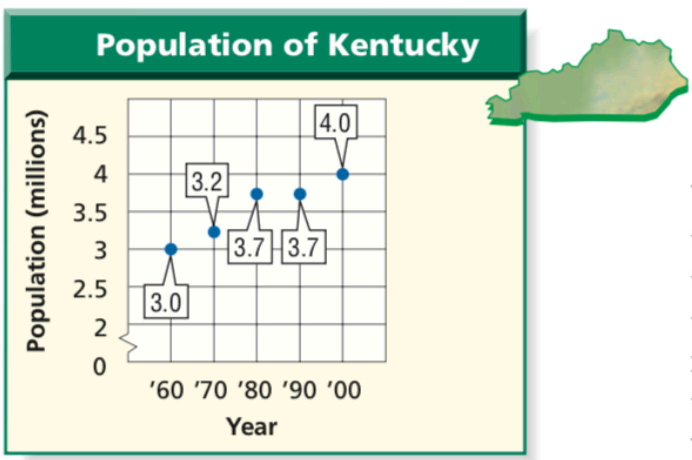
Each member of the domain is paired with exactly one member of the range, so this relation is a function.

EXAMPLE Vertical Line Test

2 **GEOGRAPHY The table shows the population of the state of Kentucky over the last several decades. Graph this information and determine whether it represents a function. Is the relation *discrete* or *continuous*?**

Year	Population (millions)
1960	3.0
1970	3.2
1980	3.7
1990	3.7
2000	4.0

Source: U.S. Census Bureau



Use the vertical line test. Notice that no vertical line can be drawn that contains more than one of the data points. Therefore, this relation is a function. Because the graph consists of distinct points, the relation is discrete.

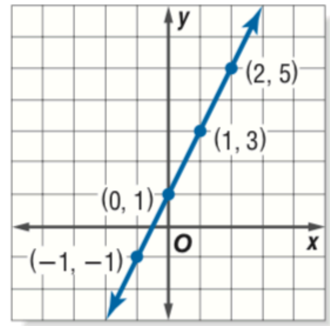
EXAMPLE Graph a Relation

3 **Graph each equation and find the domain and range. Then determine whether the equation is a function and state whether it is *discrete* or *continuous*.**

a. $y = 2x + 1$

Make a table of values to find ordered pairs that satisfy the equation. Choose values for x and find the corresponding values for y. Then graph the ordered pairs.

x	y
−1	−1
0	1
1	3
2	5



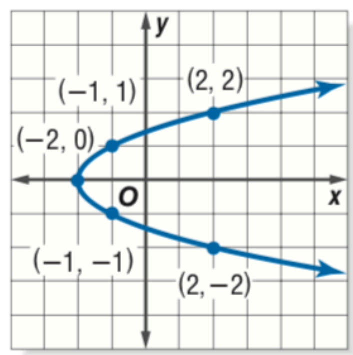
Since x can be any real number, there is an infinite number of ordered pairs that can be graphed. All of them lie on the line shown. Notice that every real number is the x-coordinate of some point on the line. Also, every real number is the y-coordinate of some point on the line. So the domain and range are both all real numbers, and the relation is continuous.

This graph passes the vertical line test. For each x-value, there is exactly one y-value, so the equation $y = 2x + 1$ represents a function.

b. $x = y^2 - 2$

Make a table. In this case, it is easier to choose y values and then find the corresponding values for x. Then sketch the graph, connecting the points with a smooth curve.

x	y
2	−2
−1	−1
−2	0
−1	1
2	2



Every real number is the y-coordinate of some point on the graph, so the range is all real numbers. But, only real numbers greater than or equal to -2 are x-coordinates of points on the graph. So the domain is $\{x | x \geq -2\}$. The relation is continuous.

You can see from the table and the vertical line test that there are two y values for each x value except $x = -2$. Therefore, the equation $x = y^2 - 2$ does not represent a function.

EXAMPLE 4 Evaluate a Function

Given $f(x) = x^2 + 2$, find each value.

a. $f(-3)$

$f(x) = x^2 + 2$ Original function

$f(-3) = (-3)^2 + 2$ Substitute.

$= 9 + 2$ or 11 Simplify.

b. $f(3z)$

$f(x) = x^2 + 2$ Original function

$f(3z) = (3z)^2 + 2$ Substitute.

$= 9z^2 + 2$ $(ab)^2 = a^2b^2$

State the domain and range of each relation. Then determine whether each relation is a function. Write *yes* or *no*.

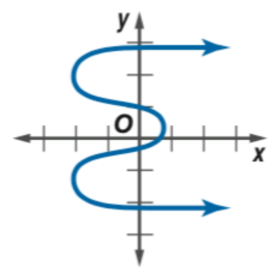
1.

Year	Population
1970	11,605
1980	13,468
1990	15,630
2000	18,140

2.

x	y
1	5
2	5
3	5
4	5

3.



Graph each relation or equation and find the domain and range. Then determine whether the relation or equation is a function and state whether *discrete* or *continuous*.

4. $\{(1, 2), (2, 3), (3, 4), (4, 5)\}$ **5.** $\{(0, 3), (0, 2), (0, 1), (0, 0)\}$

7. $y = 2x - 1$ **8.** $y = 2x^2$

Find each value if $f(x) = x + 7$ and $g(x) = (x + 1)^2$.

10. $f(2)$ **11.** $f(-4)$ **12.** $f(a + 2)$ **13.** $g(4)$

14. $g(-2)$ **15.** $f(0.5)$ **16.** $g(b - 1)$ **17.** $g(3c)$

LINEAR EQUATIONS

EXAMPLE Identify Linear Functions

1 State whether each function is a linear function. Explain.

a. $f(x) = 10 - 5x$ This is a linear function because it can be written as $f(x) = -5x + 10$. $m = -5$, $b = 10$

b. $g(x) = x^4 - 5$ This is not a linear function because x has an exponent other than 1.

c. $h(x, y) = 2xy$ This is not a linear function because the two variables are multiplied together.

Real-World EXAMPLE Evaluate a Linear Function

2 WATER PRESSURE The linear function $P(d) = 62.5d + 2117$ can be used to find the pressure (lb/ft^2) d feet below the surface of the water.

a. Find the pressure at a depth of 350 feet.

$P(d) = 62.5d + 2117$ Original function

$P(350) = 62.5(350) + 2117$ Substitute.

$= 23{,}992$ Simplify.

The pressure at a depth of 350 feet is about 24,000 lb/ft^2.

b. The term 2117 in the function represents the atmospheric pressure at the surface of the water. How many times as great is the pressure at a depth of 350 feet as the pressure at the surface?

Divide the pressure 350 feet down by the pressure at the surface.

$\frac{23{,}992}{2117} \approx 11.33$ Use a calculator.

The pressure at that depth is more than 11 times that at the surface.

EXAMPLE 3 Standard Form

Write each equation in standard form. Identify A, B, and C.

a. $y = -2x + 3$

$y = -2x + 3$ Original equation

$2x + y = 3$ Add $2x$ to each side.

So, $A = 2$, $B = 1$, and $C = 3$.

b. $-\frac{3}{5}x = 3y - 2$

$-\frac{3}{5}x = 3y - 2$ Original equation

$-\frac{3}{5}x - 3y = -2$ Subtract $3y$ from each side.

$3x + 15y = 10$ Multiply each side by -5 so that the coefficients are integers and $A \geq 0$.

So, $A = 3$, $B = 15$, and $C = 10$.

EXAMPLE 4 Use Intercepts to Graph a Line

Find the x-intercept and the y-intercept of the graph of $3x - 4y + 12 = 0$. Then graph the equation.

The x-intercept is the value of x when $y = 0$.

$3x - 4y + 12 = 0$ Original equation

$3x - 4(0) + 12 = 0$ Substitute 0 for y.

$3x = -12$ Subtract 12 from each side.

$x = -4$ Divide each side by 3.

The x-intercept is -4. The graph crosses the x-axis at $(-4, 0)$.

Likewise, the y-intercept is the value of y when $x = 0$.

$3x - 4y + 12 = 0$ Original equation

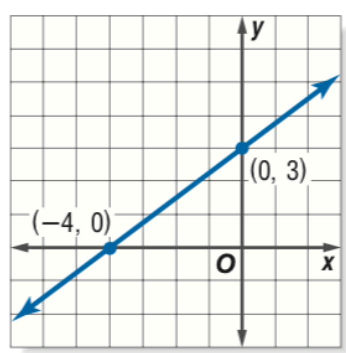
$3(0) - 4y + 12 = 0$ Substitute 0 for x.

$-4y = -12$ Subtract 12 from each side.

$y = 3$ Divide each side by -4.

The y-intercept is 3. The graph crosses the y-axis at $(0, 3)$.

Use these ordered pairs to graph the equation.



State whether each equation or function is linear. Write *yes* or *no*. If no, explain your reasoning.

1. $\frac{x}{2} - y = 7$
2. $\sqrt{x} = y + 5$
3. $g(x) = \frac{2}{x-3}$
4. $f(x) = 7$

Write each equation in standard form. Identify *A*, *B*, and *C*.

5. $x + 7 = y$

8. $y = \frac{2}{3}x + 8$
9. $-0.4x = 10$
10. $0.75y = -6$

Find the *x*-intercept and the *y*-intercept of the graph of each equation. Then graph the equation.

11. $2x + y = 6$

14. $x = 3y$
15. $\frac{3}{4}y - x = 1$
16. $y = -3$

SLOPE

EXAMPLE Find Slope and Use Slope to Graph

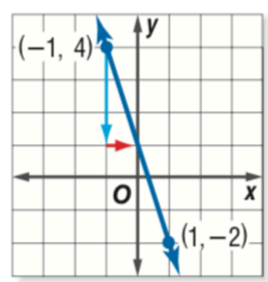
1 Find the slope of the line that passes through (–1, 4) and (1, –2). Then graph the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope formula}$$

$$= \frac{-2 - 4}{1 - (-1)} \quad (x_1, y_1) = (-1, 4), (x_2, y_2) = (1, -2)$$

$$= \frac{-6}{2} \text{ or } -3 \quad \text{The slope is } -3.$$

Graph the two ordered pairs and draw the line. Use the slope to check your graph by selecting any point on the line. Then go down 3 units and right 1 unit or go up 3 units and left 1 unit. This point should also be on the line.



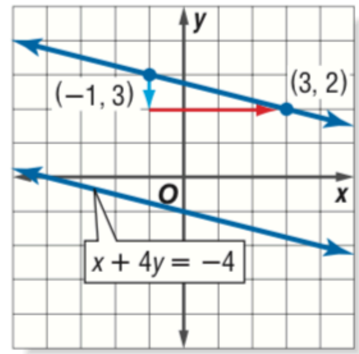
EXAMPLE Parallel Lines

3 **Graph the line through (–1, 3) that is parallel to the line with equation $x + 4y = -4$.**

The x-intercept is -4, and the y-intercept is -1. Use the intercepts to graph $x + 4y = -4$.

The line falls 1 unit for every 4 units it moves to the right, so the slope is $-\frac{1}{4}$.

Now use the slope and the point at $(-1, 3)$ to graph the line parallel to the graph of $x + 4y = -4$.



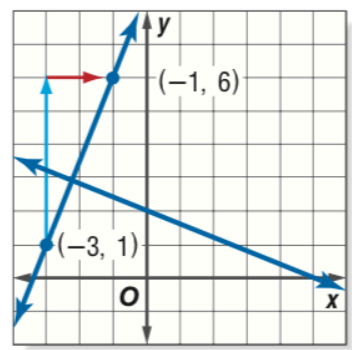
EXAMPLE Perpendicular Lines

4 **Graph the line through (–3, 1) that is perpendicular to the line with equation $2x + 5y = 10$.**

The x-intercept is 5, and the y-intercept is 2. Use the intercepts to graph $2x + 5y = 10$.

The line falls 2 units for every 5 units it moves to the right, so the slope is $-\frac{2}{5}$. The slope of the perpendicular line is the opposite reciprocal of $-\frac{2}{5}$, or $\frac{5}{2}$.

Start at $(-3, 1)$ and go up 5 units and right 2 units. Use this point and $(-3, 1)$ to graph the line.



Find the slope of the line that passes through each pair of points.

1. (0, 3), (5, 0)
2. (2, 3), (5, 7)
3. (2, 8), (2, –8)

Graph the line passing through the given point with the given slope.

7. (0, 3); 1
8. (2, 3); 0
9. (–1, 1); $-\frac{1}{3}$

Graph the line that satisfies each set of conditions.

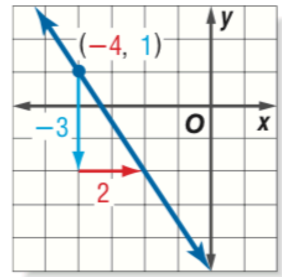
10. passes through (0, 1), parallel to a line with a slope of –2
11. passes through (4, –5), perpendicular to the graph of $-2x + 5y = 1$

WRITING LINEAR EQUATIONS

EXAMPLE Write an Equation Given Slope and a Point

1 **Write an equation in slope-intercept form for the line that has a slope of $-\frac{3}{2}$ and passes through $(-4, 1)$.**

$y = mx + b$	Slope-intercept form
$1 = -\frac{3}{2}(-4) + b$	$(x, y) = (-4, 1)$, $m = -\frac{3}{2}$
$1 = 6 + b$	Simplify.
$-5 = b$	Subtract 6 from each side.



The equation in slope-intercept form is $y = -\frac{3}{2}x - 5$.

STANDARDIZED TEST EXAMPLE Write an Equation Given Two Points

2 What is an equation of the line through $(-1, 4)$ and $(-4, 5)$?

A $y = -\frac{1}{3}x + \frac{11}{3}$ **B** $y = \frac{1}{3}x + \frac{13}{3}$ **C** $y = -\frac{1}{3}x + \frac{13}{3}$ **D** $y = -3x + 1$

Read the Test Item

You are given the coordinates of two points on the line.

Solve the Test Item

First, find the slope of the line.

$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$= \frac{5 - 4}{-4 - (-1)}$	$(x_1, y_1) = (-1, 4)$, $(x_2, y_2) = (-4, 5)$
$= \frac{1}{-3}$ or $-\frac{1}{3}$	Simplify.

Then write an equation.

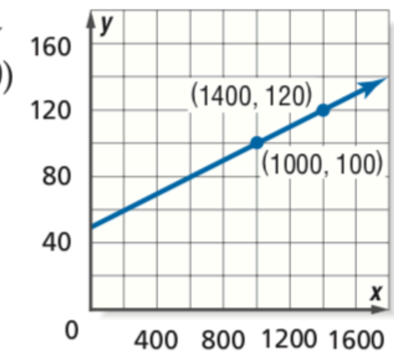
$y - y_1 = m(x - x_1)$	Point-slope form
$y - 4 = -\frac{1}{3}[x - (-1)]$	$m = -\frac{1}{3}$; use either point for (x_1, y_1).
$y = -\frac{1}{3}x + \frac{11}{3}$	The answer is A.

Real-World EXAMPLE

3 **SALES** **As a salesperson, Eric Fu is paid a daily salary plus commission. When his sales are \$1000, he makes \$100. When his sales are \$1400, he makes \$120.**

a. Write a linear equation to model this situation.

Let x be his sales and let y be the amount of money he makes. Use the points (1000, 100) and (1400, 120) to make a graph to represent the situation.



$m = \frac{y_2 - y_1}{x_2 - x_1}$ — Slope formula

$= \frac{120 - 100}{1400 - 1000}$ — $(x_1, y_1) = (1000, 100)$, $(x_2, y_2) = (1400, 120)$

$= 0.05$ — Simplify.

Now use the slope and either of the given points with the point-slope form to write the equation.

$y - y_1 = m(x - x_1)$ — Point-slope form

$y - 100 = 0.05(x - 1000)$ — $m = 0.05$, $(x_1, y_1) = (1000, 100)$

$y - 100 = 0.05x - 50$ — Distributive Property

$y = 0.05x + 50$ — Add 100 to each side.

The slope-intercept form of the equation is $y = 0.05x + 50$.

b. What are Mr. Fu's daily salary and commission rate?

The y-intercept of the line is 50. The y-intercept represents the money Eric would make if he had no sales. In other words, \$50 is his daily salary.

The slope of the line is 0.05. Since the slope is the coefficient of x, which is his sales, he makes 5% commission.

c. How much would Mr. Fu make in a day if his sales were \$2000?

Find the value of y when $x = 2000$.

$y = 0.05x + 50$ — Use the equation you found in part **a**.

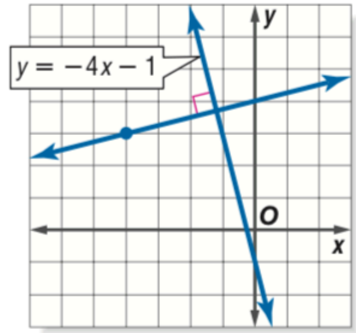
$= 0.05(2000) + 50$ — Replace x with 2000.

$= 100 + 50$ or 150 — Simplify.

Mr. Fu would make \$150 if his sales were \$2000.

EXAMPLE Write an Equation of a Perpendicular Line

4 **Write an equation for the line that passes through (–4, 3) and is perpendicular to the line whose equation is $y = -4x - 1$.**



The slope of the given line is –4. Since the slopes of perpendicular lines are opposite reciprocals, the slope of the perpendicular line is $\frac{1}{4}$.

Use the point-slope form and the ordered pair (–4, 3).

$y - y_1 = m(x - x_1)$ Point-slope form

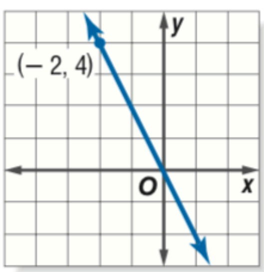
$y - 3 = \frac{1}{4}[x - (-4)]$ $(x_1, y_1) = (-4, 3), m = \frac{1}{4}$

$y - 3 = \frac{1}{4}x + 1$ Distributive Property

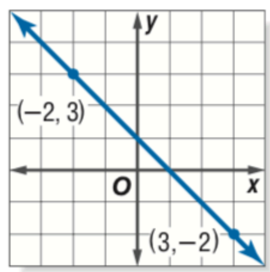
$y = \frac{1}{4}x + 4$ Add 3 to each side.

Write an equation in slope-intercept form for each graph.

1.



2.



Write an equation in slope-intercept form for the line that satisfies each set of conditions.

3. slope –1, passes through (7, 2)

4. slope $\frac{3}{4}$, passes through the origin

5. passes through (1, –3) and (–1, 2)

6. x-intercept –5, y-intercept 2

7. passes through (1, 1), parallel to the graph of $2x + 3y = 5$

8. passes through (0, 0), perpendicular to the graph of $2y + 3x = 4$

SOLVING SYSTEMS OF EQUATIONS

EXAMPLE Solve the System of Equations by Completing a Table

1 **Solve the system of equations by completing a table.**

$-2x + 2y = 4$

$-4x + y = -1$

Write each equation in slope-intercept form.

$-2x + 2y = 4 \quad \rightarrow \quad y = x + 2$

$-4x + y = -1 \quad \rightarrow \quad y = 4x - 1$

Use a table to find the solution that satisfies both equations.

x	$y_1 = x + 2$	y_1	$y_2 = 4x - 1$	y_2	(x, y_1)	(x, y_2)
-1	$y_1 = (-1) + 2$	1	$y_2 = 4(-1) - 1$	-5	$(-1, 1)$	$(-1, -5)$
0	$y_1 = 0 + 2$	2	$y_2 = 4(0) - 1$	-1	$(0, 2)$	$(0, -1)$
1	$y_1 = (1) + 2$	3	$y_2 = 4(1) - 1$	3	$(1, 3)$	$(1, 3)$

The solution of the system of equations is the ordered pair that satisfies both equations.

The solution of the system is (1, 3).

EXAMPLE Solve by Graphing

2 **Solve the system of equations by graphing.**

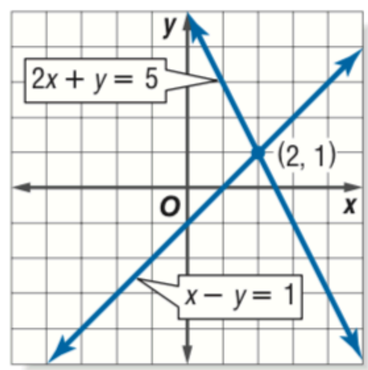
$2x + y = 5$

$x - y = 1$

Write each equation in slope-intercept form.

$2x + y = 5 \quad \rightarrow \quad y = -2x + 5$

$x - y = 1 \quad \rightarrow \quad y = x - 1$



The graphs appear to intersect at (2, 1).

CHECK Substitute the coordinates into each equation.

$2x + y = 5$	$x - y = 1$	Original equations
$2(2) + 1 \stackrel{?}{=} 5$	$2 - 1 \stackrel{?}{=} 1$	Replace x with 2 and y with 1.
$5 = 5$ ✓	$1 = 1$ ✓	Simplify.

The solution of the system is (2, 1).

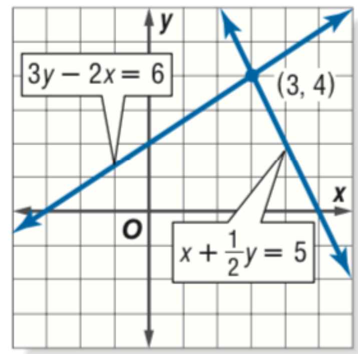
EXAMPLE Intersecting Lines

4 **Graph the system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.**

$$x + \frac{1}{2}y = 5$$

$$3y - 2x = 6$$

Write each equation in slope-intercept form.



$$x + \frac{1}{2}y = 5 \quad \rightarrow \quad y = -2x + 10$$

$$3y - 2x = 6 \quad \rightarrow \quad y = \frac{2}{3}x + 2$$

The graphs intersect at (3, 4). Since there is one solution, this system is *consistent and independent*.

EXAMPLE Same Line

5 **Graph the system of equations and describe it as *consistent* and *independent*, *consistent and dependent*, or *inconsistent*.**

$$9x - 6y = 24$$

$$6x - 4y = 16$$

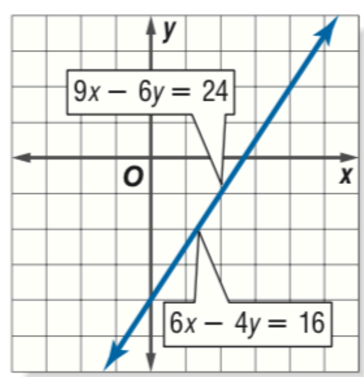
Write each equation in slope-intercept form.

$$9x - 6y = 24 \quad \rightarrow \quad y = \frac{3}{2}x - 4$$

$$6x - 4y = 16 \quad \rightarrow \quad y = \frac{3}{2}x - 4$$

Since the equations are equivalent, their graphs are the same line. Any ordered pair representing a point on that line will satisfy both equations.

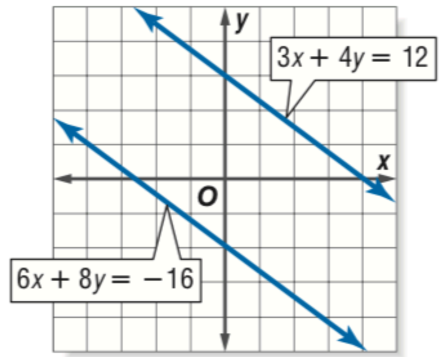
So, there are infinitely many solutions to this system. It is *consistent and dependent*.



EXAMPLE Parallel Lines

6 **Graph the system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.**

$$\mathbf{3x + 4y = 12}$$
$$\mathbf{6x + 8y = -16}$$



$$3x + 4y = 12 \quad \rightarrow \quad y = -\frac{3}{4}x + 3$$

$$6x + 8y = -16 \quad \rightarrow \quad y = -\frac{3}{4}x - 2$$

The lines do not intersect. Their graphs are parallel lines. So, there are no solutions that satisfy both equations. This system is *inconsistent*.

EXAMPLE Solve by Using Substitution

1 **Use substitution to solve the system of equations.**

$$\mathbf{x + 2y = 8}$$
$$\mathbf{\frac{1}{2}x - y = 18}$$

Solve the first equation for x in terms of y.

$x + 2y = 8$ First equation

$x = 8 - 2y$ Subtract $2y$ from each side.

Substitute $8 - 2y$ for x in the second equation and solve for y.

$\frac{1}{2}x - y = 18$ Second equation

$\frac{1}{2}(8 - 2y) - y = 18$ Substitute $8 - 2y$ for x.

$4 - y - y = 18$ Distributive Property

$-2y = 14$ Subtract 4 from each side.

$y = -7$ Divide each side by -2.

Now, substitute the value for y in either original equation and solve for x.

$x + 2y = 8$ First equation

$x + 2(-7) = 8$ Replace y with -7.

$x - 14 = 8$ Simplify.

$x = 22$

The solution of the system is $(22, -7)$.

STANDARDIZED TEST EXAMPLE **Solve by Substitution**

2 Matthew stopped for gasoline twice on a long car trip. The price of gasoline at the first station where he stopped was $2.56 per gallon. At the second station, the price was $2.65 per gallon. Matthew bought a total of 36.1 gallons of gasoline and spent $94.00. How many gallons of gasoline did Matthew buy at the first gas station?

A 17.6 **B** 18.5 **C** 19.2 **D** 20.1

Read the Item

You are asked to find the number of gallons of gasoline that Matthew bought at the first gas station.

Solve the Item

Step 1 Define variables and write the system of equations. Let x represent the number of gallons bought at the first station and y represent the number of gallons bought at the second station.

$x + y = 36.1$ The total number of gallons was 36.1.

$2.56x + 2.65y = 94$ The total price was $94.

Step 2 Solve one of the equations for one of the variables in terms of the other. Since the coefficient of y is 1 and you are asked to find the value of x, it makes sense to solve the first equation for y in terms of x.

$x + y = 36.1$ First equation

$y = 36.1 - x$ Subtract x from each side.

Step 3 Substitute $36.1 - x$ for y in the second equation.

$2.56x + 2.65y = 94$ Second equation

$2.56x + 2.65(36.1 - x) = 94$ Substitute $36.1 - x$ for y.

$2.56x + 95.665 - 2.65x = 94$ Distributive Property

$-0.09x = -1.665$ Simplify.

$x = 18.5$ Divide each side by -0.09.

Step 4 Matthew bought 18.5 gallons of gasoline at the first gas station. The answer is B.

EXAMPLE Solve by Using Elimination

3 **Use the elimination method to solve the system of equations.**

$\mathbf{4a + 2b = 15}$

$\mathbf{2a + 2b = 7}$

In each equation, the coefficient of b is 2. If one equation is subtracted from the other, the variable b will be eliminated.

$$\begin{array}{rl} 4a + 2b = 15 & \\ (-)\ \underline{2a + 2b = 7} & \\ 2a \qquad = 8 & \text{Subtract the equations.} \\ a = 4 & \text{Divide each side by 2.} \end{array}$$

Now find b by substituting 4 for a in either original equation.

$$\begin{array}{rl} 2a + 2b = 7 & \text{Second equation} \\ 2(4) + 2b = 7 & \text{Replace } a \text{ with 4.} \\ 8 + 2b = 7 & \text{Multiply.} \\ 2b = -1 & \text{Subtract 8 from each side.} \\ b = -\frac{1}{2} & \text{Divide each side by 2.} \end{array}$$

The solution is $\left(4, -\frac{1}{2}\right)$.

EXAMPLE Multiply, Then Use Elimination

4 **Use the elimination method to solve the system of equations.**

$$\mathbf{3x - 7y = -14}$$
$$\mathbf{5x + 2y = 45}$$

Multiply the first equation by 2 and the second equation by 7. Then add the equations to eliminate the y variable.

$3x - 7y = -14$ → Multiply by 2. → $6x - 14y = -28$

$5x + 2y = 45$ → Multiply by 7. → $(+)\ 35x + 14y = 315$

$$\begin{aligned} 41x &= 287 && \text{Add the equations.} \\ x &= 7 && \text{Divide each side by 41.} \end{aligned}$$

Replace x with 7 and solve for y.

$$\begin{aligned} 3x - 7y &= -14 && \text{First equation} \\ 3(7) - 7y &= -14 && \text{Replace } x \text{ with 7.} \\ 21 - 7y &= -14 && \text{Multiply.} \\ -7y &= -35 && \text{Subtract 21 from each side.} \\ y &= 5 && \text{Divide each side by } -7. \end{aligned}$$

The solution is (7, 5).

EXAMPLE Inconsistent System

5 **Use the elimination method to solve the system of equations.**

$$\mathbf{8x + 2y = 17}$$
$$\mathbf{-4x - y = 9}$$

Use multiplication to eliminate x.

$8x + 2y = 17$ → $8x + 2y = 17$

$-4x - y = 9$ → Multiply by 2. → $-8x - 2y = 18$

$$0 = 35 \quad \text{Add the equations.}$$

Since there are no values of x and y that will make the equation $0 = 35$ true, there are no solutions for this system of equations.

Solve each system of equations by graphing or by completing a table.

1. $x + 3y = 18$
$-x + 2y = 7$

2. $x - y = 2$
$2x - 2y = 10$

3. $2x + 6y = 6$
$\frac{1}{3}x + y = 1$

Solve each system of equations by using substitution.

1. $2x + 3y = 10$
$x + 6y = 32$

2. $x = 4y - 10$
$5x + 3y = -4$

3. $3x - 4y = -27$
$2x + y = -7$

Solve each system of equations by using elimination.

4. $7x + y = 9$
$5x - y = 15$

5. $r + 5s = -17$
$2r - 6s = -2$

6. $6p + 8q = 20$
$5p - 4q = -26$

SOLVING SYSTEMS OF INEQUALITIES

EXAMPLE Intersecting Regions

1 Solve each system of inequalities.

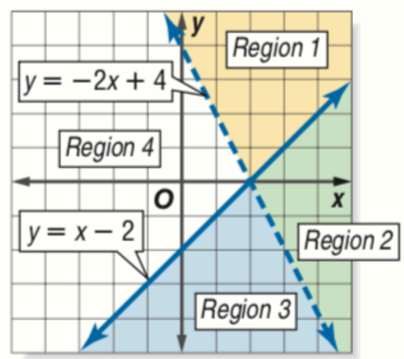
a. $y > -2x + 4$

$y \leq x - 2$

Solution of $y > -2x + 4 \rightarrow$ Regions 1 and 2

Solution of $y \leq x - 2 \rightarrow$ Regions 2 and 3

The region that provides a solution of both inequalities is the solution of the system. Region 2 is the solution of the system.

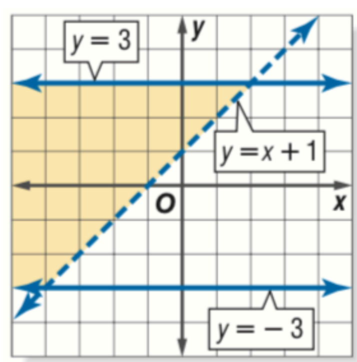


b. $y > x + 1$

$|y| \leq 3$

The inequality $|y| \leq 3$ can be written as $y \leq 3$ and $y \geq -3$.

Graph all of the inequalities on the same coordinate plane and shade the region or regions that are common to all.

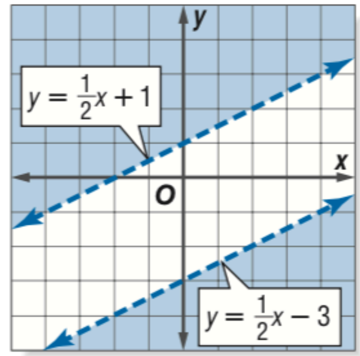


EXAMPLE 2 Separate Regions

Solve the system of inequalities by graphing.

$$y > \frac{1}{2}x + 1$$

$$y < \frac{1}{2}x - 3$$



Graph both inequalities. The graphs do not overlap, so the solution sets have no points in common. The solution set of the system is $\varnothing$.

Solve each system of inequalities.

1. $x \le 5$
   $y \ge -3$
2. $y < 3$
   $y - x \ge -1$
3. $x + y < 5$
   $x < 2$